

Cosmological Black Holes on Branes

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We examined analytically a cosmological black hole domain wall system. Using the C-metric construction we derive the metric for the spacetime describing an infinitely thin wall intersecting a cosmological black hole. We studied the behaviour of the scalar field describing a self-interacting cosmological domain wall and find the approximated solution valid for large distances. The *thin wall* approximation and the back reaction problem were elaborated finding that the topological kink solution smoothed out singular behaviour of the zero thickness wall using a core topological and hence thick domain wall. We also analyze the nucleation of cosmological black holes on and in the presence of a domain walls and conclude that the domain wall will nucleate small black holes on it rather than large ones inside.

I. INTRODUCTION

There has been recently a considerable interest in topological defects arising during phase transitions in the early universe which can play an important role in our understanding of cosmological evolution of its own right [1].

Domain walls are subject of the intense studies because of the concept that our universe might be a brane or a defect emerged in some higher dimensional geometry. This idea appears in the recent unifications attempts such as superstring theories or M-theory [2,3]. As was shown in [2] the $E_8 \times E_8$ heterotic string theory at strong coupling may be described in terms of M-theory acting in eleven-dimensional spacetime with boundaries where ten-dimensional supersymmetric Yang-Mills gauge degrees of freedom reside on two boundaries, *branes at the end of the world*. In contrast to the gauge fields acting only in a lower-dimensional submanifold gravitation can penetrate the bulk. One also hopes that this idea will be helpful to investigate the very intriguing problem of hierarchy, i.e., the huge gap between the electroweak scale and the Planck scale. Randal and Sundrum [4] proved that for nonfactorizable geometries in five-dimensional spacetime the zero modes of the Kaluza-Klein dimensional reduction can be localized in four-dimensional submanifold. They also managed to reproduce the Newtonian gravity at large distances.

Interests of cosmic string black hole systems were motivated, among other things, by the possible enlargement of the *no-hair* [5] conjecture to the problem of the nontrivial topology of field configurations surrounding black holes. In Ref. [6] the numerical and analytical evidence for Abelian Higgs vortex acting as a long hair for Schwarzschild black hole or Reissner-Nordström (RN) solution were provided. Mann and collaborators investigated the problem of vortex in the spacetime of cosmological black holes [7] while the problem of the vortex solutions in the background of an electrically charged black string was examined in [8], where it was shown that black string could also support the Abelian Higgs field as hair.

On the other hand, in dilaton gravity being the low-energy limit of the superstring theories, the problem of Abelian

Higgs vortex acting as hair on the dilaton black holes was studied in Refs. [9,10]. When the dilaton black hole tends to extremality one has always expulsion of the flux from the black hole (the so-called *Meissner effect*). This phenomenon was justified both by numerical and analytical investigations. For the vortex which is thick enough the black hole will expel its flux, causing the field to remain in its symmetric state on the event horizon. It will be not amiss to expect the similar phenomenon to occur in the case of the domain wall black hole system.

The problem of domain wall and black hole in general relativity was studied in [11] and in dilaton gravity in Ref. [12]. Among all, it was shown how to smooth this singular solution with a thick domain wall. The process of nucleation of black holes on domain walls was also tackled. In the case of dilaton gravity it was revealed analytically that the extreme dilaton black hole always expelled domain wall (we have to do with the so-called *Meissner effect*). The numerical studies of a topological domain wall and Schwarzschild black hole were conducted in [13,14]. In dilaton gravity all the analytical outcomes were confirmed numerically in [15].

Our paper is essentially concerned with the problem of a domain wall and a cosmological black hole system, providing some continuity with the previous works [9,12,15]. The paper is organized in the following way. In Sec.II we derive the infinitesimal domain wall cosmological black hole metric. Sec.III is devoted to the problem of self-gravitating cosmological domain wall. We find the approximated solution for scalar field equations describing the domain wall valid for large distances. In Sec.IV we examined the scalar field equations in the background of RN de Sitter (RN-dS) and RN-AdS spacetimes as well as in the cosmological C-metric background. The *thin wall* approximation was also described. We also tackle the problem of gravitational back-reaction. We justify that as in general relativity and dilaton gravity also in the cosmological background a topological kink smoothes out a shell-like singularity of a domain wall. In Sec.V we tackle the problem of the expulsion of domain walls from the black holes and find the approximate form for the scalar field to expulsion place place. In Sec.VI we study the production of cosmological black holes on the domain walls and nucleation of pairs of static cosmological black holes in the presence of a domain wall. We summarize and conclude our investigations in Sec.VII.

II. BLACK HOLES WITH COSMOLOGICAL CONSTANT

In this section we derive solution describing an infinitesimally thin domain wall with a cosmological black hole as the static spherically symmetric solution of Einstein equations. We begin with the action of the considered theory in the form as follows:

$$S = \int d^4x \sqrt{-g} \left[R - 2\Lambda - F_{\mu\nu}F^{\mu\nu} \right], \quad (1)$$

while equations of motion derived from the variational principle imply

$$\nabla_\mu F^{\mu\nu} = 0, \quad (2)$$

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}(F), \quad (3)$$

where the energy-momentum tensor yields

$$T_{\mu\nu}(F) = 4F_{\mu\rho}F_{\nu}{}^{\rho} - g_{\mu\nu}F^2. \quad (4)$$

In general C-metric solution has interpretation as a two oppositely charged black holes undergoing acceleration. The various aspects of the C-metric were vastly studied in literature [16]- [25].

In our studies we shall use the C-metric for cosmological black hole is as follows:

$$ds^2 = \frac{1}{A^2(x-y)^2} \left[H(y)dt^2 - \frac{dy^2}{H(y)} + \frac{dx^2}{G(x)} + G(x)d\phi^2 \right], \quad (5)$$

where

$$G(x) = a - \frac{\Lambda}{3A^2} - bx^2 - 2mAx^3 - q^2 A^2 x^4, \quad (6)$$

$$H(y) = a - by^2 - 2mAy^3 - q^2 A^2 y^4, \quad (7)$$

$\Lambda = \frac{3k}{l^2}$, $k = \pm 1$ is the cosmological constant and A is the acceleration parameter. The $U(1)$ gauge field in the magnetic and electric case are respectively $A_\phi = -qx$ and $A_t = qy$.

The freedom of a redefinition of the parameters appearing in C-metric enables one to map C-metric onto another C-metric up to a conformal transformation [16]. It can be typically eliminated by setting $a = 1$ and $b = 1$.

The roots of $G(x)$ are denoted by x_1, x_2, x_3, x_4 in ascending order, while the roots of $H(y)$ are written as y_1, y_2, y_3, y_4 . The surface $y = y_1$ is the inner black hole horizon, $y = y_2$ is the outer black hole horizon while $y = y_3$ is interpreted as the accelerated or cosmological horizon. The largest root has no physical meaning.

In order to study the cosmological constant domain wall black hole we shall use the method described in [26,27]. According to the Israel procedure [28] the discontinuity of the extrinsic curvature is provided by the tension $\tilde{\sigma}$ of a domain wall in the cosmological background. Then, it implies the following:

$$[K_{ij}] = 4\pi G \tilde{\sigma} h_{ij}, \quad (8)$$

where h_{ij} is the metric induced on the wall. Having in mind [11], an appropriate umbilic surface can be found at $x = 0$. The surface has normal $n = \frac{1}{A_y} dx$, and the induced metric may be written as

$$ds^2 = \frac{1}{A^2 y^2} \left[H(y)dt^2 - \frac{dy^2}{H(y)} + \left(1 - \frac{\Lambda}{3A^2} \right) d\phi^2 \right]. \quad (9)$$

A short calculations reveal that the extrinsic curvature in this case is equal to $K_{ij} = Ah_{ij}$ where the Israel condition implies that the domain wall tension in the cosmological background is equal to $\tilde{\sigma} = \sigma + \Lambda = A/2\pi G$. After changing the variables $r = -1/Ay$ and $T = t/A$, the metric reduces to the form as follows:

$$ds^2 = - \left(1 - \frac{2m}{r} + \frac{q^2}{r^2} - A^2 r^2 \right) dT^2 + \frac{dr^2}{\left(1 - \frac{2m}{r} + \frac{q^2}{r^2} - A^2 r^2 \right)} + r^2 \left(1 - \frac{\Lambda}{3A^2} \right) d\phi^2. \quad (10)$$

As in Ref. [11] we decide the conical singularity to lie at $x = y_3$ on the side where $x < 0$. Consequently, under this assumption if we build the domain wall black hole system by gluing two copies of each side $x > 0$, we shall get rid of the string from the spacetime.

The charge of the considered black hole can be measured by means of the integration of the flux conducted through the sphere surrounding it. It results in the form

$$Q_{BH-dw} = 2 \frac{\Delta\phi}{4\pi} \left[A_\phi(x=y_4) - A_\phi(x=0) \right] = \frac{2y_4}{qA^2(y_4-y_1)(y_4-y_2)(y_4-y_3)}. \quad (11)$$

The factor two in Eq.(11) appears because we should take into account both sides of the domain wall. The black hole built on the domain wall will neither swallow up the brane nor slide it off [29] due to the fact of existing an elastic force action exerted by the brane on the cosmological black hole.

Thus, using the C-metric construction [26,27] we have derived an equivalent of the thin cosmic string black hole solution [30], namely the metric of infinitisemally thin domain wall bisecting a cosmological black hole.

III. SELF-GRAVITATING COSMOLOGICAL DOMAIN WALL

In this section our considerations will be devoted to a general matter Lagrangian with real Higgs field and the symmetry breaking potential. The Lagrangian density for the scalar field can be written as

$$\mathcal{L}_{dw} = -\frac{1}{2}\nabla_\mu\varphi\nabla^\mu\varphi - U(\varphi). \quad (12)$$

We assume that the symmetry breaking potential $U(\varphi)$ has a discrete set of degenerate minima. The energy-momentum tensor for the scalar fields responsible for the domain wall formation implies the following:

$$T_{ij}(\varphi) = -\frac{1}{2}g_{ij}\nabla_m\varphi\nabla^m\varphi - U(\varphi)g_{ij} + \nabla_i\varphi\nabla_j\varphi. \quad (13)$$

As usual, for the convenience, we scale out parameters via transformation $X = \varphi/\eta$ and $\epsilon = 8\pi G\eta^2$. It happened that the parameter ϵ represents the gravitational strength and is connected with the gravitational interaction of the Higgs field. Let us define $V(X) = \frac{U(\varphi)}{V_F}$, where $V_F = \lambda\eta^4$. Then, we arrive at the relation:

$$8\pi G\mathcal{L}_{dw} = -\frac{\epsilon}{w^2}\left[w^2\frac{\nabla_\mu X\nabla^\mu X}{2} + V(X)\right], \quad (14)$$

where $w = \sqrt{\frac{\epsilon}{8\pi GV_F}}$ is the inverse mass of the scalar after symmetry breaking, which also characterize the width of the domain wall defect. Using Eq.(14) we can derive the relation for X field. Namely, it may be written as

$$\nabla_\mu\nabla^\mu X - \frac{1}{w^2}\frac{\partial V}{\partial X} = 0, \quad (15)$$

where $V(X) = \frac{1}{4}(X^2 - 1)^2$. In our next step we try to determine the asymptotic behaviour of X in the spacetime with cosmological constant. We shall not specify the exact sign of cosmological constant. Therefore our solution will depend on a constant $k = \pm 1$. The metric under consideration has the form:

$$ds^2 = -\left(1 - \frac{\Lambda}{3}r^2\right)dt^2 + \frac{dr^2}{\left(1 - \frac{\Lambda}{3}r^2\right)} + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (16)$$

where $\Lambda = 3k/l^2$. One can verifies that the Eq. of motion for the scalar field (15) may be written as

$$\left(1 - \frac{kr^2}{l^2}\right)X_{,rr} + \frac{2}{r}\left(1 - \frac{2kr^2}{l^2}\right)X_{,r} + \frac{1}{r^2}X_{,\theta\theta} + \frac{1}{r^2}\cot\theta X_{,\theta} - \frac{1}{w^2}\frac{\partial V}{\partial X} = 0. \quad (17)$$

Next, after changing the variables $z = r\cos\theta$ we get

$$\left(1 - \frac{kz^2}{l^2}\right)X_{,zz} - \frac{4kz}{l^2}X_{,z} - \frac{1}{w^2}X(X^2 - 1) = 0. \quad (18)$$

The exact analytic solution of Eq.(18) is not known. However if one assumes that $X(z \rightarrow \infty) \rightarrow 1$, i.e., its value tends to the value of the scalar field at large distances, we are able to examine the behaviour of its magnitude. The inspection of the equation of motion reveals that this relation is approximately satisfied by $X = X_0 \simeq 1$ for large z , where X_0 is the the minimum of the considered potential $V(X)$. This minimum is the vacuum value of the scalar field configuration. Let us assume further, that the fluctuations about the considered vacuum value is $\xi(z)$, namely

$$X \simeq X_0 + \xi(z). \quad (19)$$

Then it implies the following:

$$\frac{kz^2}{l^2} \frac{d^2\xi}{dz^2} + \frac{4kz}{l^2} \frac{d\xi}{dz} + \frac{1}{w^2} \xi(z) \simeq 0. \quad (20)$$

In the derivation of Eq.(20) we have neglected terms of order of unity in the coefficients of the first and second terms. The approximated solution for large z has the following form:

$$X_{(s-gr)} = X_0 \left[1 - \left(\frac{z}{z_0} \right)^{\frac{-3+\sqrt{9-4\beta^2}}{2}} \right], \quad (21)$$

where we have denoted by $\beta^2 = l^2/w^2k$ and z_0 is a constant of integration.

IV. DOMAIN WALLS IN THE COSMOLOGICAL CONTEXT

In this section we first demonstrate how the distributional domain wall with a cosmological black hole on it can be consistently find as a limit of physical, field theoretical topological defects. Let us consider the line element of RN-dS and RN-AdS metric which can be expressed as follows:

$$ds^2 = - \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3} r^2 \right) dt^2 + \frac{dr^2}{\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3} r^2 \right)} + r^2(d\theta^2 + \sin^2 \theta d\phi^2), \quad (22)$$

For the black hole vortex system the scalar fields were approximated as a functions of $\sqrt{g_{\phi\phi}}$ [6]. Guessing the ansatz for the scalar field in the form $X(z) = X(r \cos \theta)$ we get the expression:

$$\begin{aligned} \nabla_\mu \nabla^\mu X &= X_{,zz} \left[1 - \frac{2Mz^2}{r^3} + \frac{Q^2 z^2}{r^4} - \frac{\Lambda}{3} z^2 \right] + X_{,z} \left[\frac{Q^2 z}{r^4} - \frac{4Mz}{r^3} - \frac{5}{3} \Lambda z + \frac{1}{z} \right] \\ &= X_{,zz} - \frac{\Lambda}{3} z^2 X_{,zz} - \frac{5}{3} \Lambda z X_{,z} + \mathcal{O}\left(\frac{1}{M}\right). \end{aligned} \quad (23)$$

We remark that r is far more greater than M outside the horizon of the black hole. Then, if the thickness of the domain wall is much less smaller than the black hole horizon, i.e., $M \gg 1$ one can draw a conclusion that the scalar field X tends to its self-gravitating solution in AdS or dS spacetime depending on the sign of the cosmological constant. It can be observed that a thin wall can be painted on the cosmological black hole. This fact was also confirmed for RN and dilaton black hole solution in Refs. [11,12,15]. Our preliminary numerical studies [31] reveals that this situation also takes place for the cosmological black holes.

Because we expect that the gravitating system under consideration will have C-metric form, one can see that for the variable x is responsible $\cos \theta$ and this fact enables us to guess that $z = x/Ay$. Consequently under this assumption, one can write

$$\begin{aligned}\nabla_\mu \nabla^\mu X &= X_{,zz} \left(zA - 1 \right)^2 \left[G(x) - A^2 z^2 H(y) \right] + X_{,z} (zA - 1)^2 \left[G_{,z}(x) - z^2 A^2 H(y)_{,z} \right] \\ &\quad + X_{,z} G(x) (zA - 1) \left[-2A - 2 \frac{1}{z} + \left(\frac{zA - 1}{z^2} \right) \right] \\ &\quad + X_{,z} H(y) zA (zA - 1) \left[-A^2 z + 5A \right].\end{aligned}\tag{24}$$

Using the *thin wall* approximation [11] i.e., we expect the black hole horizon is to be large ($A | y_2 | \ll 1$) and considering the regime of large acceleration radius ($A | y_3 | \ll 1$), one concluded that

$$\nabla_\mu \nabla^\mu X = X_{,zz} - \frac{\Lambda}{3} z^2 X_{,zz} - \frac{5}{3} \Lambda z X_{,z} + \mathcal{O}(A^2).\tag{25}$$

Thus, we have arrived at the conclusion that $X = X_{(s-gr)}$ is a good approximation solution of the field equations in the cosmological C -metric spacetime.

The other interesting problem we shall take up a problem of the gravitational back reaction for the thick domain wall and cosmological black holes described by the spacetime metric (5). As in Ref. [11,12] we denote for simplicity $\Omega = A(x - y)$ and we use linearized calculations in $\epsilon = 3/2A$, writing $\Omega = \Omega_0 + A\Omega_1$ and so on. In the vicinity of the core of the domain wall we have $\Omega_1/\Omega_0 = \mathcal{O}(1)$, while in the region far away from the core it tends to zero.

From Eqs. of motion for gauge field, namely $\partial_x [\partial_x A_\phi] = 0$ and the form of the first order perturbed solution determined by $A_\mu^{(1)} = f(z) A_\mu^{(0)}$ one has that $f(z)$ is constant. Thus, the gauge field potential is unaltered by the presence of the domain wall.

As was remarked in [11,12] since the fact that variation of the extrinsic curvature due to the domain wall is carried by Ω , we guess that $G(x)$ and $H(y)$ will take effectively their background values. In this case, after some lengthy calculations one reaches to the following equations of motion:

$$R_\phi^\phi - R_x^x = -2G(x)A^2y \left(Az - 1 \right) f_{,xx} = -\epsilon \left(X_{(s-gr),z} \right)^2,\tag{26}$$

$$R_t^t - R_y^y = 2H(y)A^2y \left(Az - 1 \right) f_{,yy} = \mathcal{O}(A^2),\tag{27}$$

$$\begin{aligned}R_t^t - R_\phi^\phi &= \frac{\Omega}{2} \left[\Omega G(x)'' + \Omega H(y)'' - 2 \left(H(y)' \Omega_{,y} + G(x)' \Omega_{,y} \right) \right] \\ &= \frac{Ay}{2} \left(Az - 1 \right)^2 \left[G(x)'' + H(y)'' \right] - A^3 y^2 \left(Az - 1 \right)^2 \left[H(y)' (f_{,y} - 1) + G(x)' f_{,x} \right] = -4q^2 \Omega^4.\end{aligned}\tag{28}$$

As in Ref. [11] the relations for Ricci tensors suggest that Ω may be presented as follows:

$$\Omega = A(f - y),\tag{29}$$

where $f_0 = |x|$.

Because of the fact that $f_{,x} = \mathcal{O}(A)$ and $f_{,y} = \mathcal{O}(A^2)$ we draw a conclusion that the Einstein's equations in the cosmological context will be satisfied to the leading order in A . Substituting the above ansatz for Ω and the ansatz for the scalar field as a solution for self-gravitating scalar field in the spacetime with non-vanishing cosmological constant, we arrive at the following expression:

$$f(x) = -\epsilon A^2 y^2 X_0^2 \frac{(-5 + \sqrt{9 - 4\beta^2})^2}{4(-4 + \sqrt{9 - 4\beta^2})(-3 + \sqrt{9 - 4\beta^2})} \left(\frac{x}{Ay z_0} \right)^{-3 + \sqrt{9 - 4\beta^2}}.\tag{30}$$

From the above back reaction considerations one can draw a conclusion that as in the domain wall black hole system in general relativity [11], and domain wall dilaton black hole system in the low energy string theory [12] in the case of cosmological black hole domain wall system a kink solution also smoothes out the shell-like singularity of the infinitesimal domain wall.

V. EXPULSION OF THE DOMAIN WALL BY THICK BRANE

A. General considerations

In this section we shall confine our attention to the problem of expelling thick domain walls from the cosmological black holes. Namely, the topological defect ceases to penetrate the cosmological black hole, which is equivalent to the case that the domain wall's scalar field $X = 0$ over the black hole event horizon. The previous considerations for *thin walls* approximation indicate that for large mass black holes there is a solution with the wall intersecting black hole but it gives no indication for small mass cosmological black holes.

We shall give here the bound on cosmological black hole radius r_{BH} for which the domain wall scalar field X must be expelled from the cosmological black hole. First, let us suppose that there is a solution of the Eq. of motion for scalar field that penetrates the horizon of black hole. Thus, Eq.(15) on the horizon of the black hole becomes

$$X_{,\theta\theta} + \cot\theta X_{,\theta} - r_{BH}^2 \frac{X(X^2 - 1)}{w^2} = 0. \quad (31)$$

As we take the third derivative of (31), then one obtains the following:

$$X_{,\theta\theta\theta} = -\cot\theta X_{,\theta\theta} + X_{,\theta} \left[\frac{1}{\sin^2\theta} + r_{BH}^2 \left(\frac{3X^2}{w^2} - 1 \right) \right]. \quad (32)$$

The nontrivial solution satisfied the condition $X(\pi/2) = 0$ which caused that $X_{,\theta\theta}(\pi/2) = 0$. Let us suppose that $X_{,\theta}(\pi/2) > 0$ so that for $X_{,\theta}$ one has maximum (minimum) at $\frac{\pi}{2}$ for $r_{BH}^2 > w^2$ ($r_{BH}^2 < w^2$), because from Eq.(32) one can see that at any turning point of $X_{,\theta}$ we get

$$X_{,\theta\theta\theta} = X_{,\theta} \left[\frac{1}{\sin^2\theta} + r_{BH}^2 \left(\frac{3X^2}{w^2} - 1 \right) \right] - \cot\theta X_{,\theta\theta} > X_{,\theta} \left[1 - r_{BH}^2 \right]. \quad (33)$$

For $r_{BH}^2 < w^2$ any turning point of $X_{,\theta}$ is a minimum which is consistent with $X_{,\theta} = 0$ at $\theta = 0, \pi$. Therefore for $r_{BH}^2 < w^2$ the only possible solution is $X = 0$ on the horizon. In this case one has the expulsion.

Now we treat the second case when $r_{BH}^2 > w^2$. When a stable expelling solution takes place one has that in the vicinity of the black hole horizon it monotonically relaxes to a kink solution as we move away from the horizon. thus $X_{,\theta}$ is of the same sign (without loss of generality we take that $X_{,\theta} > 0$ for our condition). Let us consider the situation that in the vicinity of the black hole horizon one has $r = r_{BH} + \beta$, where β is a small parameter. By virtue of this Eq. of motion takes the form as follows:

$$X_{,\theta\theta} = -\cot\theta X_{,\theta} - \left(r_{BH}^2 g^{rr} X_r \right)_{,r} - r_{BH}^2 \frac{X}{w^2} + \mathcal{O}(\beta). \quad (34)$$

Consider next the higher derivative of scalar field X . One can show that $X_{,\theta\theta}, X_{,\theta\theta\theta} \geq 0$ and $X_{,\theta\theta\theta\theta} \leq 0$ on $[\frac{\pi}{2}, \pi]$ for $r_{BH}^2 > w^2$. For since we have the following:

$$\frac{\pi}{4} X_{,\theta} \left(\frac{\pi}{2} \right) < X \left(\pi \right) < \frac{\pi}{2} X_{,\theta} \left(\frac{\pi}{2} \right), \quad (35)$$

$$\frac{\pi}{4} |X_{,\theta\theta} \left(\frac{\pi}{2} \right)| < X_{,\theta} \left(\pi \right) < \frac{\pi}{2} |X_{,\theta\theta} \left(\frac{\pi}{2} \right)|. \quad (36)$$

Having in mind relation (34) and the above inequalities we obtain

$$\frac{\pi^2 r_{BH}^2}{16 w^2} X \left(\pi \right) < X \left(\pi \right) < \frac{\pi^2 r_{BH}^2}{4 w^2} X \left(\pi \right), \quad (37)$$

i.e., $\frac{2w}{\pi} < w < r_{BH} < \frac{4w}{\pi}$.

Therefore for $r_{BH} > 4w/\pi$ there is no expulsion of scalar field from the cosmological black hole interior.

The next problem we shall consider is a cosmological black hole sitting inside the domain wall. Inside the domain wall the potential terms are negligible comparing to the gradient ones. This fact justifies neglecting it. We provide an ansatz for the scalar field in the form $X(r, \theta) = b(r) \cos \theta$. Plugging this ansatz into the scalar equation of motion gives

$$\frac{d}{dr} \left[r^2 V(r) \frac{db}{dr} \right] - 2b(r) = 0, \quad (38)$$

where $V(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3} r^2$. To overcome the difficulty in solving Eq.(38) in terms of known functions we shall approximate $V(r)$ near the cosmological black hole horizon by the relation

$$V(r) \sim 2\kappa_{BH}(r - r_{BH}), \quad (39)$$

where κ_{BH} is the surface gravity of the black hole. Further, we define new variables

$$\eta^2 = \frac{r - r_{BH}}{2\kappa_{BH} r_{BH}^2}. \quad (40)$$

It can be easily verified that the relation (38) yields

$$\frac{d^2 b}{d\eta^2} + \frac{1}{\eta} \frac{db}{d\eta^2} - 8b = 0. \quad (41)$$

The solution of it may be expressed in the form

$$b = AI_0 \left(2\sqrt{2}\eta \right) + BK_0 \left(2\sqrt{2}\eta \right), \quad (42)$$

where I_0, K_0 are modified Bessel functions and A, B are constant.

Let us suppose that we have expulsion on the horizon of the black hole and find the form of scalar field in this case. On the horizon of the black hole one has $X|_{r=r_{BH}} = 0$. Moreover, we know the asymptotic behaviour of the scalar field at infinity. On this simplifications the scalar field can be brought to the form as follows:

$$X(r, \theta) \sim \sqrt{2}(r - r_{BH})^{1/4} e^{-\psi(r - r_{BH})^{1/2}} I_0 \left(\psi(r - r_{BH})^{1/2} \right) \cos \theta, \quad (43)$$

where $\psi = 2/r_{BH}(\kappa_{BH})^{1/2}$.

B. Nariai and anti-Nariai black hole domain wall systems

Now we turn our attention to the electromagnetic extension of the Nariai solution introduced by Bertotti and Robinson [32]. In Refs. [20,33] it has been concluded that a limiting approach would take near extreme RN-dS solution into the charged Nariai one. Here by the extreme solution one means the situation when the cosmological and the outer black hole horizons coincide. The charge Nariai metric is given by [25]

$$ds^2 = \frac{R_0^2}{K_0} \left(-\sin^2 \gamma d\tau^2 + d\gamma^2 \right) + R_0^2 \left(d\theta^2 + \sin^2 \theta d\phi^2 \right), \quad (44)$$

where γ and θ both run from 0 to π and ϕ has the period 2π while R_0 and K_0 are positive constants. K_0 satisfies the additional condition $0 < K_0 \leq 1$. The magnetic and electric charges are given respectively as $F_{\theta\phi} = q$ and $F_{\tau\gamma} = q/K_0$. The following redefinition of the coordinates

$$\sin^2 \gamma = 1 - \frac{K_0}{R_0} R^2, \quad \tau = \sqrt{\frac{K_0}{R_0^2}} T, \quad (45)$$

enables us to write the metric as

$$ds^2 = -N(R)dT^2 + \frac{dR^2}{N(R)} R_0^2 \left(d\theta^2 + \sin^2 \theta d\phi^2 \right), \quad (46)$$

where $N(R) = 1 - \frac{K_0}{R_0^2} R^2$. The cosmological constant Λ and the charge are related by the conditions

$$\Lambda = \frac{1 + K_0}{2R_0^2}, \quad q^2 = \frac{1 - K_0}{2} R_0^2. \quad (47)$$

The equation of motion for X field implies

$$\partial_R(N(R)\partial_R X) + \frac{1}{R_0^2} X_{,\theta\theta} + \frac{1}{R_0^2} \cot \theta X_{,\theta} = \frac{1}{w^2} X(X^2 - 1). \quad (48)$$

Suppose, that we have a flux expulsion on the horizon and try to find possible conditions for it occurs [12]. Just, on the horizon we have $X = 0$ and $N(R) \rightarrow 0$ near the black hole horizon while $R_0^2 X^3 \ll 1$. Having all these in mind and we integrate relation (48) on the interval $(\theta, \frac{\pi}{2})$, for $\theta > \beta_0$. It reveals the following:

$$\partial_\theta X(\theta) > \frac{1}{w^2} \cot \theta X R_0^2. \quad (49)$$

Next using the fact that $X_{,\theta\theta} < 0$ on $[0, \frac{\pi}{2}]$ we deduce that

$$X_{,\theta\theta} < \frac{X(\theta) - X(\theta_0)}{\theta - \theta_0} < \frac{X(\theta)}{\theta - \theta_0} < \frac{X(\theta)}{\theta - \beta}, \quad (50)$$

which in turn enables us to write the relation

$$\frac{1}{R_0^2} > \frac{1}{w^2} (\theta - \beta) \cot \theta. \quad (51)$$

The above inequalities must hold over the range of $\theta \in (\beta, \frac{\pi}{2})$ if the expulsion takes place. Because of the fact that $\theta - \beta > 0$, $\cot \theta$ on this interval is greater than zero, then the relation (51) always holds and one gets the expulsion of the thin domain wall from the charged Nariai black hole.

Applying the same considerations to the anti-Nariai black hole which the line element has the form [25]

$$ds^2 = -N(R)dT^2 + \frac{dR^2}{N(R)}R_0^2 \left(d\theta^2 + \sinh^2 \theta d\phi^2 \right), \quad (52)$$

where $N(R) = -1 + \frac{K_0}{R_0^2}R^2$ and

$$\Lambda = -\frac{1+K_0}{2R_0^2} < 0, \quad q^2 = \frac{K_0-1}{2}R_0^2, \quad (53)$$

the previous arguments can be repeated leading finally to the inequality

$$\frac{1}{R_0^2} > \frac{1}{w^2}(\theta - \beta) \coth \theta. \quad (54)$$

For the range of $\theta \in (\beta, \frac{\pi}{2})$ the relation always holds revealing the fact that as in charged Nariai case we have also the expulsion of the thin domain walls from charged anti-Nariai black holes.

Summing it all we have derived a bound on the cosmological black hole radius for which the expulsion occurred. Studying a cosmological black hole sitting inside a domain wall we found the approximate solution for the equation of motion for scalar field X . We also showed that both in the case of Nariai and anti-Nariai black holes one had to do with the complete expulsion of the domain wall's scalar fields from these black holes (the so-called *Meissner effect*).

VI. NUCLEATION OF COSMOLOGICAL BLACK HOLES ON DOMAIN WALLS

In this section the problem of nucleation of cosmological black holes on the domain walls will attract our attention. It turned out that most topological defects as strings, domain walls are equipped with tension that caused them to be unstable to snapping or forming black holes on them. One can say about two phenomena which can occur in the presence of domain wall, i.e., nucleating of a black hole on domain wall or nucleation of black holes enclosed by the domain wall.

First we shall shed some light in the presence of domain wall on the process of forming the cosmological black holes on domain walls. The probability of the process is given by $\exp[-(I - I_0)]$, where I_0 is the Euclidean action of the initial configuration. On the other hand, I stands for the Euclidean action of the final state with a domain wall and a cosmological black hole on it. The exponent in question can be viewed as the ratio of probabilities of nucleation of cosmological black holes on the domain wall. In order to build the domain wall-black hole instanton we shall follow the footsteps described in [34]. Then it follows that we have

$$I = -\frac{1}{4} \left(\mathcal{A}_{acc} + \mathcal{A}_{BH} \right), \quad (55)$$

where \mathcal{A}_{acc} is the area of cosmological (accelerated horizon), \mathcal{A}_{BH} is the area of the black hole horizon. Consequently, Eq.(55) yields the result that

$$I = -\frac{2\pi y_4}{A^2 |G'(y_4)|} \left[\frac{1}{(y_3 - y_4)y_3} + \frac{1}{(y_2 - y_4)y_2} \right]. \quad (56)$$

The Euclidean action for a domain wall can be expressed as $I_0 = -1/8\pi\sigma^2$ [35]. The probability of this process will be more transparent in the limit of small black holes for which $M, Q \ll 1/A$. The resulting result implies

$$I = -\frac{1}{8\pi\sigma^2} + \frac{M}{\sigma} + \frac{3Mk}{l^2\sigma^2} + \mathcal{O}(A^2). \quad (57)$$

Thus, one gets

$$I - I_0 = (I - I_0)_{RN-dw} + \frac{3Mk}{l^2\sigma^2}, \quad (58)$$

where $(I - I_0)_{RN-dw}$ describes the value of the exponent for nucleation rate of RN black hole domain wall system. Here the mass of a black hole is a parameter which can be varied independently of the wall tension. This fact envisages that it can be arbitrary small.

The problem of production of neutral and charged pairs of black holes was intensively studied in literature [35] while cosmological pairs black holes production was elaborated in Ref. [21]. For the completeness of discussion and reader's convenience we briefly comment on these results.

The general situation comprises constructing a two-sided bubble by taking two regions of appropriate spacetime and joining them along the boundary satisfying the Israel matching conditions. We shall pay attention to nucleation of static black holes, i.e., black holes which attractive gravitational energy exactly counterbalanced the repulsive energy of domain walls. The matching conditions may be given by $\sqrt{V(r) - \dot{r}} = 2\pi\sigma r$. The above Eq. can be interpreted as the motion of a fictitious particle in a potential $v = V(r) - (2\pi\sigma r)^2$. On the other hand, the static solution for which $r = r_{st}$ has zero energy and it is obtained by solving the above matching condition under the supplementary condition of the form $\frac{\partial v}{\partial r} = 0$.

The Euclidean action for the instanton is given by

$$I_E = -\frac{1}{16\pi} \int_{M_E} d^4x \sqrt{g} \left(R - 2\Lambda - F_{\mu\nu}F^{\mu\nu} \right) - \frac{\sigma}{2} \int_W d^3x \sqrt{h}, \quad (59)$$

where Euclidean section M_E includes the values on both sides of the domain wall. After some further reductions in which we make use of Einstein's Eqs. relation (59) can be brought to the form

$$I_E = \beta_{BH} \left[-2\pi\sigma \sqrt{V(r_{st})} r_{st}^2 + Q^2 \left(\frac{1}{r_{BH}} - \frac{1}{r_{st}} \right) - \frac{k}{\pi l^2} \left(r_{st}^3 - r_{BH}^3 \right) \right], \quad (60)$$

where β_{BH} is the black hole period in imaginary time $\tau = -it$. The probability for the pairs creation of a static cosmological black holes in the vicinity of a domain wall will be given by $\exp(-I_E)$.

In contrast to the process of production of cosmological black holes on the domain walls the nucleation in the presence of a domain wall is characterized by the fixed mass [21,36] depending on the value of the domain wall tension σ . For it cannot be varied independently of the domain wall's tension a certain (large size) black holes can be nucleated. Hence, this process will be heavily suppressed.

VII. CONCLUSIONS

In this paper we have studied the problem of cosmological black holes on the domain walls. By means of the C-metric construction presented in [26,27] we derived the metric for the infinitisemally thin domain wall intersecting a cosmological black hole. We also examined the problem of self-interacting cosmological domain wall and found the approximated solution for the scalar field valid for large distances outside it. We analyzed the scalar field equation

of motion in the background of RN-AdS, RN-dS and C-metric finding the *thin wall* approximation which was used in examining the back reaction problem. As in general relativity [11] and in dilaton gravity [12,15] we justified that the the *singular* behaviour of the zero thickness domain wall could be smoothed out by a core of a topological (thick) domain wall. We analyzed the domain wall cosmological black hole system and gave analytical arguments for the domain wall's scalar fields to be expelled from the cosmological black hole. Only for the charged Nariai and anti-Nariai black holes we analytically revealed the expulsion of scalar fields (the analog of the *Meissner effect*). Of course, there are preliminary analytic studies. Due to the complications of equations of motion for scalar field in other interesting backgrounds numerical studies should be conducted. Work in this direction is in progress [31].

We compared two processes of nucleation of cosmological black holes on the domain wall and the pairs productions of the black holes in the presence of the domain wall. It happened that the mass parameter of black holes on the domain walls could be carried independently to the domain wall surface density thus could be arbitrary small. In contrast to this phenomenon the nucleation of pairs of cosmological black holes in the presence of domain wall is heavily suppressed due to the fact that a certain large size black holes can be produced. The black hole's mass depends on the domain wall tension. Summing it all up one can draw a conclusion that domain wall will prefer to nucleate small black holes on it rather than large one inside it. The extension of these problems to the situations in which charged domain walls or supersymmetric domain walls and dilaton couplings are included remains an interesting open question. We hope to return to it elsewhere.

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